

An Explanation for the Slopes of Stellar Cusps in Galaxy Spheroids

Philip F. Hopkins^{1*} & Eliot Quataert¹

¹*Department of Astronomy and Theoretical Astrophysics Centre, University of California Berkeley, Berkeley, CA 94720*

Submitted to MNRAS, September, 2010

ABSTRACT

The stellar surface mass density profiles at the centers of typical $\sim L_*$ and lower-mass spheroids exhibit power law “cusps” with $\Sigma \propto R^{-\eta}$, where $0.5 \lesssim \eta \lesssim 1$ for radii $\sim 1 - 100$ pc. Observations and theory support models in which these cusps are formed by dissipative gas inflows and nuclear starbursts in gas-rich mergers. At these comparatively large radii, stellar relaxation is unlikely to account for, or strongly modify, the cuspy stellar profiles. We argue that the power-law surface density profiles observed are a natural consequence of the gravitational instabilities that dominate angular momentum transport in the gravitational potential of a central massive black hole. The dominant mode at these radii is an $m = 1$ lopsided/eccentric disk instability, in which stars torquing the gas can drive rapid inflow and accretion. Such a mode first generically appears at large radii and propagates inwards by exciting eccentricities at smaller and smaller radii, where $M_*(< R) \ll M_{\text{BH}}$. When the stellar surface density profile is comparatively shallow with $\eta < 1/2$, the modes cannot efficiently propagate to $R = 0$ and so gas piles up and star formation steepens the profile. But if the profile is steeper than $\eta = 1$, the inwards propagation of eccentricity is strongly damped, suppressing inflow and bringing η down again. Together these results produce an equilibrium slope of $1/2 \lesssim \eta \lesssim 1$ in the potential of the central black hole. These physical arguments are supported by nonlinear numerical simulations of gas inflow in galactic nuclei. Together, these results naturally explain the observed stellar density profiles of “cusp” elliptical galaxies.

Key words: galaxies: active — galaxies: evolution — quasars: general — galaxies: nuclei — galaxies: bulges — cosmology: theory

1 INTRODUCTION

Observations have established that typical $\lesssim L_*$ ellipticals and bulges exhibit steep central “cusps” in their surface luminosity density and stellar mass density profiles – i.e. a continued rise in a power-law like fashion towards small radii (Lauer et al. 1991, 1992; Crane et al. 1993; Ferrarese et al. 1994; Kormendy et al. 1994; Lauer et al. 1995; Kormendy 1999). Faber et al. (1997) showed that power-law nuclear profile ellipticals also tend to have higher degrees of rotational support and diskyness. This, together with other observations (Kormendy 1999; Quillen et al. 2000; Rest et al. 2001; Lauer et al. 2007; Ferrarese et al. 2006; Côté et al. 2007), has supported the idea that the cusp ellipticals are the direct product of gas-rich mergers and nuclear star formation during such mergers. Quantitatively, power-law cusps have

$$I \propto \Sigma_* \propto R^{-\eta} \quad (1)$$

with $0.5 \lesssim \eta \lesssim 1$ representing the typical observed slopes; the power-law profile extends from the smallest radii observed in nearby spheroids (~ 1 pc) to anywhere from ~ 10 to ~ 100 pc (Ferrarese et al. 1994; Kormendy et al. 2009).¹ The most massive spheroids deviate from this behavior and exhibit flattened nuclear profiles, or “cores.” This is, however, widely believed to be due to “scouring” by a binary black hole in a gas-poor environment (see e.g. Begelman et al. 1980) and thus does not reflect the initial formation history of the central stars that we focus on here.

Barnes & Hernquist (1991) and Mihos & Hernquist (1994) showed in simulations that tidal torques in mergers can drive rapid gas inflows, providing the fuel to power intense nuclear starbursts and build up the central stellar surface densities (see also Kormendy 1999; Hopkins et al. 2009). This is observed in local Ultraluminous Infrared Galaxies (ULIRGs), whose nuclei constitute the most rapidly star-forming environments in the local Universe. Moreover, the observed central gas densities and star formation rates in ULIRGs will leave them with typical power-law like cusps when the starburst is complete (Hibbard & Yun 1999; Tacconi et al. 2002).

Despite this progress, no theoretical explanation exists for *why* spheroid cusps should have a power law-like form in the range observed. The scales of the observed cusps are comparable to, or less than, the black hole (BH) radius of influence, and the potential is thus quasi-Keplerian. At these radii, stars almost certainly formed primarily dissipatively in a gas-rich disk, rather than via violent relaxation of a pre-existing stellar disk. On the very smallest scales, two-body relaxation is expected to drive the system to a Bahcall & Wolf (1976) cusp; however outside of ~ 1 pc the relaxation time is $\gg t_{\text{Hubble}}$. Moreover, the fact that the observed central cusps are disky and elliptical, often with significant rotational support (see references above) suggests that two-body relaxation has not had a large effect. Instead, an understanding of the observed stellar cusps appears to require the combined effects of angular momentum redistribution and star formation in disky, gas+stellar+BH systems.

Recently, we have shown that the formation of lopsided, eccentric disks within the BH radius of influence is a ubiquitous feature in hydrodynamic simulations of massive gas inflows in galaxies; such lopsided disks lead to efficient angular momentum trans-

* E-mail: phopkins@astro.berkeley.edu

¹ Provided it is defined over the same dynamic range, this is non-parametric and the same logarithmic slopes are recovered when Sersic fits are used.

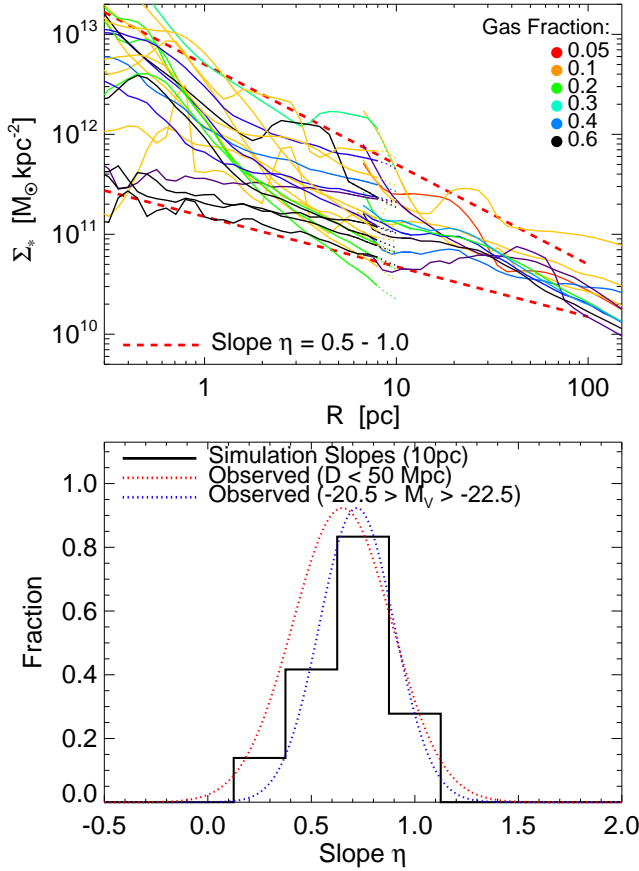


Figure 1. *Top:* Nuclear stellar mass profiles (cusps) produced in hydrodynamic simulations of gas inflow driven by gravitational instabilities in dissipative starbursts; a lopsided/eccentric disk mode dominates at $\lesssim 100$ pc. Simulations of different resolution/spatial scale are each shown only over the modeled dynamic range; the highest resolution simulations extend to ~ 100 pc while more typical simulations begin at ~ 10 pc. The simulations, taken from Hopkins & Quataert (2010a), have a wide range of initial mass profiles, gas fractions, and stellar feedback models. However, in all cases the simulated mass profiles are consistent with cusps having power law-like slopes in the range $0.5 \lesssim \eta \lesssim 1$ ($\Sigma \propto R^{-\eta}$; dashed line shows these two cases, which roughly bracket the simulations). *Bottom:* Distribution of cusp slopes. The slopes are measured in each simulation at 10 pc (fitting a power law to each from 3 – 30 pc or the maximum dynamic range allowed within this interval), and from the observations in Lauer et al. (2007) at approximately the same radius in both volume-limited and magnitude-limited sub-samples (showing the “cusp,” as opposed to “core” component of the observed bimodal distribution).

fer from the gas to the stars, powering BH accretion rates of up to $\sim 10 M_{\odot} \text{ yr}^{-1}$ (Hopkins & Quataert 2010a). Moreover, the stellar relics of these disks are reasonably similar to the nuclear disks observed on $\lesssim 10$ pc scales around nearby supermassive BHs (Hopkins & Quataert 2010b), particularly the well-studied case at the center of M31 (Lauer et al. 1993). There are also many candidate nuclear disks observed in other systems (Lauer et al. 1996, 2005; Houghton et al. 2006; Thatte et al. 2000; Debattista et al. 2006; Afanasiev & Sil’chenko 2002; Seth et al. 2010; Ledo et al. 2010).

Figure 1 (top panel) shows the stellar surface density profiles at the end of the “nuclear-scale” and “ultra-high” resolution simulations of Hopkins & Quataert (2010a), which extend inwards from $\gtrsim 10 - 100$ pc with ~ 0.1 pc resolution; we show results in the quasi steady state phase of all simulations with significant inflows,

$\geq 0.3 M_{\odot} \text{ yr}^{-1}$ into < 1 pc, sustained for $> 10^5$ yr. These SPH simulations include gas, stars, star formation, and a black hole as an additional collisionless particle; the simulations are idealized problems focused on studying the nonlinear evolution of gravitationally unstable systems in the potential of a massive black hole. Hopkins & Quataert (2011) show that the central dynamics and inflows are dominated by the nuclear $m = 1$ modes. In Figure 1 the absolute stellar mass densities depend on the initial conditions (e.g., total gas mass), but the slopes are more robust; the simulations shown span a wide range in initial gas fractions, prescriptions for star formation and gas physics, initial stellar and gas mass profiles, and bulge-to-disk ratios (see Tables 1-3 in Hopkins & Quataert 2010a), but converge to similar slopes. Comparing with the observed power law slopes of ellipticals (bottom panel), the agreement is reasonable. In this *Letter*, we provide a physical explanation for these results.

2 PROPAGATION OF INSTABILITIES

Physically, the lopsided or eccentric disk mode (azimuthal wavenumber $m = 1$ or amplitude $\propto \cos \phi$) is unique in any nearly Keplerian potential (Tremaine 2001). Gravitational torques from other modes are suppressed by the gravity of the BH. However, the resonant response between the epicyclic and orbital frequencies allows for global, low frequency $m = 1$ modes that can exert strong torques on the gas by inducing orbit crossing and shocks (e.g., Chang et al. 2007; Hopkins & Quataert 2011). Because of the importance of the $m = 1$ modes for redistributing gas inside the potential of the BH, we now focus on the physics of these $m = 1$ modes, in particular their propagation to smaller radii.

2.1 The WKB Limit

Consider an initially axisymmetric, thin, planar disk (surface density Σ) with a BH of mass M_{BH} at the coordinate center; we use cylindrical coordinates throughout (R, ϕ, z). The initial potential in the disk plane can be written $\Phi_0 = \Phi_0(R)$, and other properties are defined in standard terms:

$$V_c^2 = R \frac{\partial \Phi}{\partial R} \approx \frac{GM_{\text{enc}}(< R)}{R} \quad (2)$$

$$\Omega \equiv \iota_{\text{dyn}}^{-1} = V_c / R \quad (3)$$

$$\kappa^2 \equiv R \frac{d\Omega^2}{dR} + 4\Omega^2 = \frac{\partial^2 \Phi}{\partial R^2} + 3\Omega^2 \quad (4)$$

where V_c is the circular velocity, Ω the angular velocity, and κ the epicyclic frequency. We use c_s to denote the sound speed in a gaseous disk and σ_z the vertical dispersion in a stellar disk.

We consider a linear perturbation $\Sigma \rightarrow \Sigma_0(R) + \Sigma_1(R, \phi)$ (where Σ is the total gas+stellar disk surface density) in a frame rotating with the perturbation pattern speed Ω_p , and decompose the perturbation into linearly independent modes:

$$\Sigma_m \equiv \Sigma_a(R) \exp \{i(m\phi - \omega t)\} \quad (5)$$

$$\Sigma_a(R) \equiv |a(R)| \Sigma_0(R) \exp \left\{ i \int^R k dR \right\} \quad (6)$$

where m is the azimuthal wavenumber, $|a| = |a(R)|$ the effective mode amplitude, k the radial wavenumber, and the complex ω the mode frequency. With these definitions, the mode pattern speed is $\Omega_p \equiv \text{Re}(\omega)/m$, and the mode growth rate $\gamma \equiv \text{Im}(\omega)$.

We adopt a power-law disk as a convenient reference model:

$$\Sigma \propto R^{-\eta} = \Sigma_0 \left(\frac{R}{R_0} \right)^{-\eta} \quad (7)$$

It is straightforward to show then that

$$\Omega^2 = \frac{GM_{\text{BH}}}{r^3} + \frac{2\pi\alpha G\Sigma_0}{R_0} \left(\frac{R}{R_0}\right)^{-(\eta+1)} \quad (8)$$

where $\alpha = (\Gamma[1 - \frac{\eta}{2}]\Gamma[\frac{1+\eta}{2}]) / (\Gamma[\frac{3-\eta}{2}]\Gamma[\frac{\eta}{2}])$ for $0 < \eta < 2$.

We first consider modes in the WKB limit of tight-winding (i.e. local modes), where $|kR| \gg m$. We caution that this limit does not, in fact, hold for many of the global modes of most interest, but it is nevertheless instructive. We follow Tremaine (2001)'s derivation for a slow mode ($\Omega_p \ll \Omega$) in which the non-Keplerian part of the potential is small, i.e. $\Phi = \Phi_{\text{BH}} + \Phi_d$ where $\Phi_d/\Phi_{\text{BH}} \sim M_d/M_{\text{BH}} \ll 1$. Expanding the equations of motion in terms $\mathcal{O}(\Phi_d/\Phi_{\text{BH}})$ gives the WKB dispersion relation (to leading order in $|kR|^{-1}$) of quasi-Keplerian slow modes,

$$\omega = \varpi + \pi G\Sigma_d |k| \Omega^{-1} - c_s^2 k^2 \Omega^{-1} \quad (9)$$

for a gas disk, or

$$\begin{aligned} \omega &= \varpi + \pi G\Sigma_d |k| \Omega^{-1} \mathcal{F} \\ &\approx \varpi + \pi G\Sigma_d |k| \Omega^{-1} \exp(-\beta |kR|) \end{aligned} \quad (10)$$

for a stellar disk, where we define

$$\varpi \equiv \frac{\Omega^2 - \kappa^2}{2\Omega} = -\frac{1}{2\Omega} \left(\frac{2}{r} \frac{d}{dr} + \frac{d^2}{dr^2} \right) \Phi_d.$$

In the dispersion relation for a stellar disk (eq. 10), \mathcal{F} is the standard reduction factor (Binney & Tremaine 1987), and the latter equality in equation 10 is a convenient approximation for softened gravity, with $\beta \approx \sigma_z/V_c \approx h/R$ (the stellar disk scale height).

The $m = 1$ slow modes are stable in the limit $M_d \ll M_{\text{BH}}$ (Tremaine 2001). Because of this physical constraint, the $m = 1$ modes first appear at large radii – the radius $\equiv R_{\text{crit}}$ where $M_d/M_{\text{BH}} \sim 1$, i.e. where the potential is transitioning to Keplerian (Hopkins & Quataert 2010a; Hopkins 2010). The pattern speed Ω_p of the unstable mode is $\sim \Omega(R_{\text{crit}})$. But if the mode can propagate inwards at constant Ω_p , it will eventually be a slow mode, relative to the local Ω at smaller radii.

How does this propagation occur? The wave packets propagate with approximate group velocity $\sim V_c(R_{\text{crit}})$, so the timescale for the mode to travel is just the dynamical time at R_{crit} . However, if these modes are forming in realistic “initial” disks, and if they are the dominant source of angular momentum transport, then the initial disk surface density profile cannot already be steep. Dimensionally, $\varpi \sim (1/R\Omega)d\Phi_e/dR \sim G\Sigma/\Omega R$ (exactly true if the disk has a locally power-law profile; $\varpi = -\alpha(2-\eta)\pi G\Sigma/\Omega R$). Since Ω diverges $\propto r^{-3/2}$ at small radii, if the surface density profile is sufficiently shallow (and the dispersion is finite) then the right-hand side of Equation 9 becomes arbitrarily small as $r \rightarrow 0$, and finite ω cannot be supported – the wave will refract back at some minimum radius R_{min} . From Equation 9, this Q -barrier occurs when

$$|\Omega_p| \geq \varpi + \frac{\pi^2}{4} \frac{(G\Sigma_d)^2}{c_s^2 \Omega} \quad (11)$$

in gas or

$$|\Omega_p| \geq \varpi + \frac{\pi G\Sigma}{e\beta R\Omega} \approx \varpi + \frac{\pi G\Sigma}{e\sigma_z} \quad (12)$$

in stars. Since $\varpi \sim G\Sigma/\Omega R \propto R^{1/2-\eta}$ at small radii, for systems with finite c_s or constant β and a shallow $\Sigma \propto R^{-\eta}$ with $\eta \lesssim 1/2$, the initial waves cannot reach $R = 0$. Note that Ostriker et al. (1992) show that the same restriction applies for modes in a pure fluid disk with a hard outer edge.

If the slope is too shallow, but the $m = 1$ modes are present, they will drive gaseous inflows that will “pile up” near the refraction radius. This will steepen the mass profile and increase the self-gravity at this radius, eventually allowing further mode propagation (in both gas and stars). Once $\eta \gtrsim 1/2$, then the RHS of Equation 9 no longer vanishes as $r \rightarrow 0$, and the modes can propagate through to $R = 0$. Physically, the propagation can be understood as the eccentric mode at larger radii exciting strong eccentric perturbations at smaller radii.

Consider two nearly-adjacent annuli at radii R' and R_1 : the material at R_1 is part of the $m = 1$ mode, the material at $\leq R'$ remains unperturbed. In the WKB limit the mode behavior at larger radii is swamped by the nearest asymmetric term – i.e. just inside R_1 , the perturbing potential is $\approx \Phi_1(R_1) = 2\pi G\Sigma_1(R_1)|k|^{-1}$ (since there is no local corrugation to cancel this out). In the global limit the result is similar: at small radii inside an eccentric ring at radius R_1 having mass M_{ring} and $m = 1$ amplitude $|a|$, the magnitude of the local perturbed potential is just $\approx |a|GM_{\text{ring}}/R_1 \sim \pi G\Sigma_1(R_1)R_1$, i.e., the same as the WKB result with $|k| \sim R_1^{-1}$. To estimate the velocity induced at smaller radii by this perturbed potential, we note that for a cold gas or stellar disk, the local pattern speed of the $m = 1$ mode is just $\omega - \varpi \approx \pi G\Sigma_0 |k| \Omega^{-1}$. Together, this leads to the result that the response (in both gas and stars) at smaller radii $\sim R'$ to the eccentric disk at larger radii is given by: $|v_r/V_c| \sim (\Sigma_1/\Sigma_0)|kR|^{-1} \sim |a|$. In other words, for a non-negligible mode amplitude $|a| \sim \Sigma_1/\Sigma_0$ and a global mode with $|kR| \sim 1$, large eccentricities and hence large coherent $m = 1$ mode amplitudes, can be induced. The induced modes at these somewhat smaller radii can, in turn, excite large coherent eccentricities in the material at yet smaller radii, and so on, allowing the perturbation to grow even at arbitrarily small R .

The above derivation also implies, however, that there is a regime in which the inwards propagation of eccentricity will be inefficient. For a global mode at R_1 , the perturbed potential is $\sim \pi G\Sigma_1(R_1)R_1$, so the response $|e| \propto \Sigma_1(R_1)R_1/\Sigma(R)R = |a(R_1)|\Sigma(R_1)R_1/\Sigma(R)R$. For a sufficiently flat mass profile $\Sigma(R_1)R_1 > \Sigma(R)R$, i.e., $\eta < 1$ for $\Sigma(R) \propto R^{-\eta}$, the induced perturbation is large down to arbitrarily small R . But if the mass profile is too steep, $\Sigma(R_1)R_1 < \Sigma(R)R$, or $\eta > 1$, then although the mode can formally be supported even at small radii, the induced amplitude will decline as one moves to $R \rightarrow 0$. Crudely, we expect the propagation efficiency defined as $\log(|a(R)|/|a(R_1)|)$ to decline $\propto -\eta$, for $\eta > 1$. This is explicitly demonstrated for a very large sample of models by Zakamska & Tremaine (2004) in the context of eccentricity propagation from an external perturber to the inner planets in planetary systems (the cutoff at $\eta < 1/2$ is not evident in this approach, however, because of the discrete nature of the problem). If $\eta > 1$ is established by some means, the low efficiency of eccentricity propagation to small radii implies that gas will not inflow as efficiently at small radii. This will flatten the gas density profile and star formation will do the same for the stellar density profile, providing a mechanism for the system to self-adjust to have $\eta \lesssim 1$.

2.2 Linear Global Modes

The WKB results above are not exact, especially when the modes of interest are global and the disk mass is significant relative to M_{BH} (both of which are typically the case!). To show that our conclusions are robust, we also demonstrate the same points regarding the propagation of modes using exact linear solutions for particular global normal modes. Our methodology is described in detail in Hopkins (2010), which we briefly summarize here. We define R_0 for the power-law disk model (eq. 7) so that $M_d(<$

$R_0) = M_{\text{BH}}/[\alpha(2-\eta)]$ – i.e. $\Sigma_0 = M_{\text{BH}}/(2\pi\alpha R_0^2)$. Unlike in the WKB analysis we do not expand the equations to linear order in M_d/M_{BH} but keep the full linear perturbation equations. We consider a stellar-dominated (collisionless) disk since in our numerical results, the dominant torques in the gas are due to the stellar modes. The resulting equations of motion for linear perturbations are

$$\begin{aligned} 0 = & -(\Omega - \omega) \Sigma_a \\ & + \frac{\Sigma}{r^2 \Delta} [2\Omega(\nu_\Sigma + \nu_\Omega - \nu_\Delta) - (\Omega - \omega)] \Phi_a \\ & + \frac{\Sigma}{r \Delta} \left[(\Omega - \omega)(1 + \nu_\Sigma - \nu_\Delta) + \Omega(2 + \nu_\Omega) - \frac{\kappa^2}{2\Omega} \right] \Phi'_a \\ & + \frac{\Sigma}{\Delta} [\Omega - \omega] \Phi''_a \end{aligned} \quad (13)$$

where $\Phi_a = \int_0^\infty dr' r' P(r, r') \Sigma_a(r')$ follows from Poisson's equation,² $\nu_X \equiv \partial \ln X / \partial \ln R$, and $\Delta \equiv \kappa^2 - (\Omega - \omega)^2$.

It is straightforward to solve eq. 13 for the eigenfunctions (normal modes) of the system. For convenience and realism we modify the disk mass profiles with a steep outer power-law cut-off ($\Sigma \propto R^{-\eta} (1 + [R/a]^2)^{-(3-\eta)/2}$) so that they have finite total mass $M_d = M_{\text{BH}}$; Hopkins (2010) shows that the exact choice of M_d and/or the cutoff radius has no effect on any of our conclusions.

Figure 2 shows some of the resulting normal modes for a stellar disk. The growth rates γ and pattern speeds Ω_p are indicated on each panel, in units of $\Omega(R_0)$. For any choice of disk parameters, there is a large variety of normal modes; here, we focus on the most rapidly growing "global" modes in each model. The results are qualitatively similar for all global modes (local modes, potentially supported at all radii but localized in radius, are not of interest here). We take $\beta = 0.1$ for the softening, but our conclusions are essentially identical for a wide range of β ; in Hopkins (2010) we show that this extends to $\beta \gtrsim 0.3$, i.e. nearly-spherical configurations. This is because the manner in which $\Sigma(R)$ enters the equations means that the important dimensional parameter is really $M_{\text{enc}}(< R)$ at a given radius, so puffer systems and even multiple overlapping disks making a quasi-spherical configuration give a qualitatively identical result. The important parameter we focus on here is the power-law index of the disk mass profile η , for which we show various choices in Figure 2: $\eta = 0.05, 0.50, 0.80, 1.10, 1.85$.

For each value of η , Figure 2 shows the absolute value and real component of the surface density perturbation $a(R) = \Sigma_a/\Sigma$ and the induced eccentricity R_a/R , where R_a is the magnitude of the radial perturbation from the linear equations of motion. The modes are normalized so that $\text{MAX}(|a(R)|) = 1$. Where the eccentricities are significant, there can be orbit crossings and shocks in the gas. This dissipation helps drive rapid inflow; see Hopkins & Quataert (2011) for a detailed discussion of this physics.

The key result in Figure 2 is how the structure of the global modes changes with η ; this confirms our intuition derived from the WKB approximation. When the disk surface density profile is shallow ($\eta \lesssim 1/2$), the modes cannot propagate inwards efficiently –

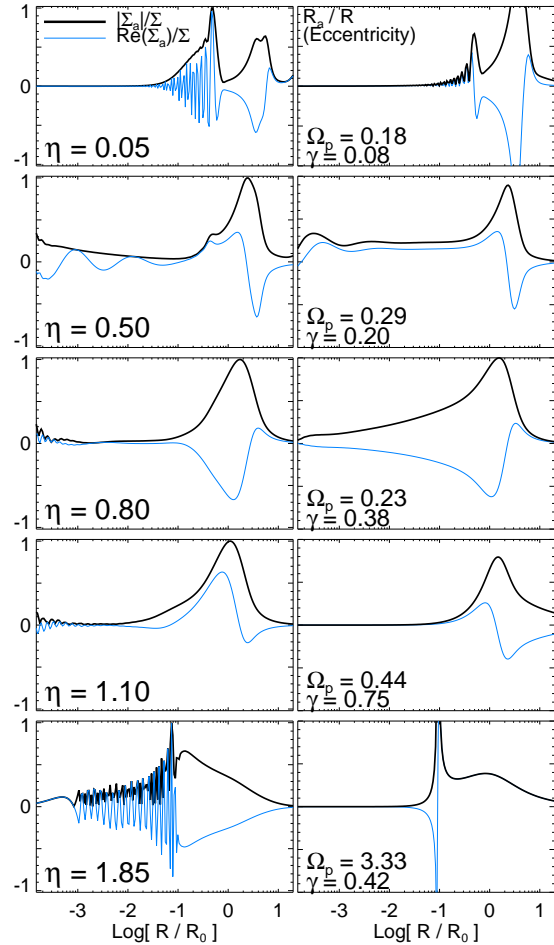


Figure 2. Global linear normal modes for gravitational perturbations to a nuclear stellar disk around a BH; the total disk-to-BH mass ratio = 1. Each row highlights one mode for a mass profile slope $\Sigma \propto R^{-\eta}$ and scale height $\beta \sim h/R = 0.1$; the pattern speed Ω_p and growth rate γ of the mode are labeled on the right in units of $\Omega(R_0)$. Radii are in units of R_0 , the BH radius of influence, $\sim 10 - 100$ pc. *Left:* Mode amplitude: black is absolute value, blue is $\text{Re}(\Sigma_a/\Sigma)$. *Right:* Real and absolute value of the induced eccentricity. Shallow surface density profiles ($\eta < 1/2$) cannot support modes at small R and so are spatially localized. For larger η , the modes propagate to $R \rightarrow 0$. For $\eta > 1$, however, the inwards propagation of eccentricity is less efficient (the disk is “stiffer” against external perturbations), which would suppress shocks and gas inflow at small R . The characteristic profile shapes can therefore self-adjust to have $1/2 \lesssim \eta \lesssim 1$.

they are confined to a moderate range of radii. At $\eta = 1/2$, the modes are suddenly able to propagate to arbitrarily small R . Going to somewhat larger $\eta = 0.7 - 0.8$, the structure of the modes is quite similar. For $\eta > 1$, the induced eccentricity is strongly suppressed at small radii (see § 2.1) even though the mode formally has non-zero amplitude to arbitrarily small R . As a consequence, the induced gas inflow would also be strongly suppressed.

3 DISCUSSION AND CONCLUSIONS

The general interpretation of the stellar density profiles of $\lesssim L_*$ ellipticals is that violent relaxation produces the outer “wings” of the mass profile, while a nuclear starburst similar to that observed in nearby ULIRGs and many high-redshift galaxies produces a dense stellar relic that dominates the mass profile inside the central $\sim \text{kpc}$ (e.g., Faber et al. 1997). The mass profile at smaller radii is thus set

² The kernel P is defined by $P(r, r') = -\pi G b_{1/2}(r_{<}/r_{>})/r_{>} + \pi G r/r'^2$ and includes the direct and indirect components of the potential, respectively. The Laplace coefficient $b_{1/2}$ is given by

$$b_{1/2}(x) = \frac{2}{\pi} \int_0^\pi \frac{\cos \theta d\theta}{(1 - 2x \cos \theta + x^2 + \beta^2)^{1/2}} \quad (14)$$

where β represents the gravitational softening, as it appeared in the WKB approximation. For $\beta > 0$, the disk potential and Ω are slightly modified accordingly, but this is a minor effect.

by the physics of angular momentum redistribution and star formation. This inner dissipative region is no longer sensitive to large-scale torques such as are produced in a merger. Instead, the inflow is likely due to secondary gravitational instabilities that develop as the disk component of gas and stars becomes self-gravitating on small scales (Shlosman et al. 1989).

In previous work (Hopkins & Quataert 2010a) we have demonstrated that inside a radius from one to several times the BH radius of influence, the character of the instabilities that dominate angular momentum redistribution changes: the non-axisymmetry is dominated by an eccentric/lopsided disk or one-armed spiral mode – a “slow” $m = 1$ mode, unique to quasi-Keplerian potentials.

If the stellar and gaseous density profiles are relatively shallow (e.g., as might be the case absent an earlier epoch of gas inflow) the $m = 1$ modes cannot be supported down to $R \rightarrow 0$, but reflect off of a Q -boundary at a finite radius. Between this inner boundary and co-rotation, however, the modes will drive accretion, steepening the gas density profile; star formation will steepen the corresponding stellar density profile. As the mass profile steepens, the $m = 1$ modes can propagate deeper in the potential, until a critical slope is reached, at which point the modes can propagate to, and drive inflow to, $R = 0$; for a power-law disk with $\Sigma \propto R^{-\eta}$, the critical slope is $\eta = 1/2$.

If gas is driven to small radii very efficiently, star formation will likely ensure that both the gas and stellar mass profiles further steepen. The surface density profile can, however, eventually become sufficiently steep that the inwards propagation of eccentricity is inefficient: the outer asymmetric perturbation is weak compared to the local disk self-gravity at small radii. For a power-law disk with $\Sigma \propto R^{-\eta}$, this occurs at $\eta = 1$. The resulting pile-up of mass at larger radii, together with continued star formation, will flatten the gas and stellar mass profiles.

The net result is a plausible equilibrium: in the presence of significant gas inflow from larger radii, star formation and the propagation of $m = 1$ modes will self-adjust so that the surface density profile satisfies $1/2 \lesssim \eta \lesssim 1$ inside the potential of the central black hole. Remarkably, this range of slopes is comparable to what is observed in the centers of “cusp” ellipticals (Fig. 1).

On the smallest scales near the BH, dynamical relaxation plays an important role in setting the stellar density profile (see Bahcall & Wolf 1976). Such effects are, however, unlikely to be important outside of ~ 1 pc because the N -body relaxation time becomes long compared to the Hubble time. The presence of massive perturbers can significantly accelerate stellar relaxation, but it is unclear how disturbed the nuclei of these (now) gas-poor galaxies are. Moreover, the observational evidence for disk structures in “cusp” galaxies suggests weak/incomplete relaxation, which would have to be very extreme to operate out to 10 – 100 pc in any case. Even a merger of binary black holes tends to leave these structures if it is gas-rich (Hopkins & Quataert 2010a), because most of the nuclear gas inflows tend to follow the BH binary coalescence, regenerating the disk, and multiple overlapping generations of disks can be formed (which will given the same mass profile, since each individually must satisfy the same $\Sigma(R)$ or $M_{\text{disk}}(R)$ scaling). And our conclusions are robust to a number of variations in e.g. the disk thickness and detailed structure. In the potential of the BH, relaxation can be enhanced by the very same resonance between orbital and epicyclic motion that is so critical for the presence of $m = 1$ modes (Kocsis & Tremaine 2010). Scalar resonant relaxation, which modifies the eccentricity axes of stellar orbits, is, however, inefficient at the radii of interest. Vector resonant relaxation is likely to be important, but this only changes the inclination angles of the disk orbits;

this may wash out some of the observable diskyness or introduce warps into the nuclear kinematics, but it will not significantly affect the mass profiles.

The combined effects of star formation and gas inflow driven by $m = 1$ modes provide a plausible explanation for the stellar mass profiles of “cusp” ellipticals at $\lesssim 10 - 100$ pc. This explanation links such cuspy profiles to the physics of angular momentum transport and BH growth. It also makes observational predictions: (1) the characteristic radii of those slopes should be correlated with the radii of influence of the BH (or radii enclosing comparable mass); (2) non-negligible radial anisotropy should be difficult to remove from the stellar orbits; (3) similar profile shapes should be observable in late-stage merger remnants, provided sufficient new stars have formed in the central regions.

ACKNOWLEDGMENTS

We thank Scott Tremaine for useful conversations. Support for PFH was provided by the Miller Institute for Basic Research in Science, University of California Berkeley.

REFERENCES

- Afanasiev, V. L., & Sil’chenko, O. K. 2002, *A&A*, 388, 461
- Bahcall, J. N., & Wolf, R. A. 1976, *ApJ*, 209, 214
- Barnes, J. E., & Hernquist, L. E. 1991, *ApJL*, 370, L65
- Begelman, M. C., Blandford, R. D., & Rees, M. J. 1980, *Nature*, 287, 307
- Binney, J., & Tremaine, S. 1987, *Galactic dynamics* (Princeton, NJ, Princeton University Press, 1987)
- Chang, P., Murray-Clay, R., Chiang, E., & Quataert, E. 2007, *The Astrophysical Journal*, 668, 236
- Côté, P., et al. 2007, *ApJ*, 671, 1456
- Crane, P., et al. 1993, *AJ*, 106, 1371
- Debatista, V. P., Ferreras, I., Pasquali, A., Seth, A., De Rijcke, S., & Morelli, L. 2006, *ApJL*, 651, L97
- Faber, S. M., et al. 1997, *AJ*, 114, 1771
- Ferrarese, L., van den Bosch, F. C., Ford, H. C., Jaffe, W., & O’Connell, R. W. 1994, *AJ*, 108, 1598
- Ferrarese, L., et al. 2006, *ApJS*, 164, 334
- Hibbard, J. E., & Yun, M. S. 1999, *ApJL*, 522, L93
- Hopkins, P. F. 2010, *MNRAS*, in press, arXiv:1009.4702 [astro-ph]
- Hopkins, P. F., Cox, T. J., Dutta, S. N., Hernquist, L., Kormendy, J., & Lauer, T. R. 2009, *ApJS*, 181, 135
- Hopkins, P. F., & Quataert, E. 2010a, *MNRAS*, 407, 1529
- . 2010b, *MNRAS*, 405, L41
- . 2011, *MNRAS*, 415, 1027
- Houghton, R. C. W., Magorrian, J., Sarzi, M., Thatte, N., Davies, R. L., & Krajnović, D. 2006, *MNRAS*, 367, 2
- Kocsis, B., & Tremaine, S. 2010, *MNRAS*, in press, arXiv:1006.0001
- Kormendy, J. 1999, in *Astronomical Society of the Pacific Conference Series*, Vol. 182, *Galaxy Dynamics - A Rutgers Symposium*, ed. D. R. Merritt, M. Valluri, & J. A. Sellwood, 124–+
- Kormendy, J., Dressler, A., Byun, Y. I., Faber, S. M., Grillmair, C., Lauer, T. R., Richstone, D., & Tremaine, S. 1994, in *Dwarf Galaxies* (Garching: ESO), ed. G. Meylan & P. Prugniel, 147–+
- Kormendy, J., Fisher, D. B., Cornell, M. E., & Bender, R. 2009, *ApJS*, 182, 216
- Lauer, T. R., et al. 1991, *ApJL*, 369, L41

- . 1992, *AJ*, 104, 552
- . 1993, *AJ*, 106, 1436
- . 1995, *AJ*, 110, 2622
- . 1996, *ApJL*, 471, L79+
- . 2005, *AJ*, 129, 2138
- . 2007, *ApJ*, 664, 226
- Ledo, H. R., Sarzi, M., Dotti, M., Khochfar, S., & Morelli, L. 2010, *MNRAS*, 407, 969
- Mihos, J. C., & Hernquist, L. 1994, *ApJL*, 431, L9
- Ostriker, E. C., Shu, F. H., & Adams, F. C. 1992, *ApJ*, 399, 192
- Quillen, A. C., Bower, G. A., & Stritzinger, M. 2000, *ApJS*, 128, 85
- Rest, A., van den Bosch, F. C., Jaffe, W., Tran, H., Tsvetanov, Z., Ford, H. C., Davies, J., & Schafer, J. 2001, *AJ*, 121, 2431
- Seth, A. C., et al. 2010, *ApJ*, 714, 713
- Shlosman, I., Frank, J., & Begelman, M. C. 1989, *Nature*, 338, 45
- Tacconi, L. J., Genzel, R., Lutz, D., Rigopoulou, D., Baker, A. J., Iserlohe, C., & Tecza, M. 2002, *ApJ*, 580, 73
- Thatte, N., Tecza, M., & Genzel, R. 2000, *A&A*, 364, L47
- Tremaine, S. 2001, *AJ*, 121, 1776
- Zakamska, N. L., & Tremaine, S. 2004, *AJ*, 128, 869